Propagators on the two-dimensional light-cone

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Abstract

Light-cone quantization procedure recently presented is applied to the two-dimensional light-cone theories. By introducing the two distinct null planes it is shown that the modification term in the two-dimensional massless light-cone propagators suggested about twenty years ago vanishs.

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In spite of the lack of manifest Lorentz covariance the light-cone gauge (radiation gauge in light-cone coordinate) has been frequently used for the calculation of perturbative QCD and the quantization of the supersymmetric Yang-Mills theories.[1] Also it has been used for the noncovariant formulation of string theories.[2]

However when the perturbation is calculated in the light-cone gauge, there is a subtle point for the prescription of the "spurious" singularity. If the usual principal-value(PV) prescription which plays an important role in other non-covariant gauges is choosed, the perturbation gives a poorly defined integral.[3] In order to escape the difficulty the new prescription which is usually called Mandelstam-Leibbrandt(ML) prescription[4] is suggested. Later the ML-prescription is also derived in Ref.[5], in the framework of equal-time canonical quantization and the renormalizibility of the gauge theories formulated in this way is proved[6] in spite of the appearance of an infinite number of nonlocal divergent term.

However this prescription is not directly applicable to calculation based on the light-cone coordinate. Recently it is verified that the ML-prescription is also derived in the light-cone coordinate when one treats the relevant degrees of freedom carefully.[7] In Ref[7] it was shown that in order to recover the ML-prescription a characteristic surface $x^+ = 0$ must be used to initialize the unphysical fields. In addition when the Poincaré charge is calculated, not only the usual $x^+ = 0$ surface but also the boundary wings must be included to the hypersurface where the energy-momentum tensor is integrated. More recently it is shown that two distinct null planes are required for the implementation of the ML-prescription in the interacting theories.[8]

About twenty years ago a similar problem with ML- and PV-prescriptions

was already discussed in the two-dimensional light-cone coordinates.[9] The authors in Ref.[9] suggested that the propagators must be modified as follows in the two-dimensional light-cone by analyzing the massless scalar and fermion theories

$$\frac{1}{p^2 + i\epsilon} \Rightarrow \frac{1}{p^2 + i\epsilon} + \frac{i\pi}{2} \frac{\delta(p^+)}{\mid p^- \mid}. \tag{1}$$

This suggestion is closely related with the choice of the prescriptions in the light-cone gauge theories. To see this by using

$$ML\left(\frac{1}{p^+}\right) = PV\left(\frac{1}{p^+}\right) - i\pi\epsilon(p^-)\delta(p^+) \tag{2}$$

where ML and PV mean the correspondent prescriptions, Eq.(1) is simply written as

$$\frac{1}{2p^-}ML\left(\frac{1}{p^+}\right) \Rightarrow \frac{1}{2p^-}PV\left(\frac{1}{p^+}\right). \tag{3}$$

This means that the "spurious" singularity problem occurs not only in the gauge choice but also in the coordinate choice. The purpose of this short paper is to apply the method developed in Ref.[7, 8] to the two-dimensional massless theory and to show that the modification term in Eq.(1) is cancelled by introducing the two distinct null planes. However this does not guarantee that the ML-prescription is more consistent than PV-prescription. The only thing we can say is that these two prescriptions describe a different theories in two dimension. Of course there can be various suggestion to determine which is more consistent prescription in the light-cone coordinates. The one way is to calculate the vacuum expectation value(VEV) of a Wilson-loop operator defined on the null-planes. Recently it is shown that the result of VEV of Wilson-loop operator does not coincide with that of the 't Hooft approach at two-loop level[10] if the ML-prescription is choosed.[11]

So it seems to be important to perform the same calculation with the PV-prescription and compare the result with that of the 't Hooft approach.

In order to show the disappearance of the modification term appeared in Eq.(1), let us start with free scalar Lagrangian[12]

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi. \tag{4}$$

Lagrangian (4) gives the equation of motion

$$\partial_+ \partial_- \phi = 0 \tag{5}$$

and the energy-momentum tensor

$$T^{++} = \partial_- \phi \partial_- \phi, \tag{6}$$

$$T^{--} = \partial_+ \phi \partial_+ \phi,$$

$$T^{+-} = T^{-+} = 0.$$

If we take a Dirac procedure with a primary constraint

$$\Omega = \Pi_\phi - \partial_- \phi \tag{7}$$

where Π_ϕ is a canonical momentum of ϕ with respect to the time coordinate x^+ and define a Green's function as

$$G(x,y) = i < 0 \mid T^+ \phi(x)\phi(y) \mid 0 > \tag{8}$$

where T^+ is x^+ - ordered product, then we obtain the result (1). In order to use the two distinct null planes we consider the general solution of Eq.(5)

$$\phi(x) = u(x^+) + v(x^-). \tag{9}$$

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If we add the boundary wings to the hypersurface σ when the Poincaré charge is calculated by the formula

$$P^\mu = \int_\sigma T^{\mu\nu} d\sigma_\nu, \tag{10}$$

the Poincaré charges become

$$P^+ = \int_{-\infty}^{\infty} dx^- \left[\partial_- v(x^-) \partial_- v(x^-) \right]_{x^+ = 0} \tag{11}$$

$$P^- = \int_{-\infty}^{\infty} dx^+ \left[\partial_+ u(x^+) \partial_+ u(x^+) \right]_{x^- = 0}.$$

The difference of this approach from Ref.[9] is the non-zero of P^-. From the fact that the Heisenberg equation leads the equations of motion

$$\partial_- u = 0, \tag{12}$$

$$\partial_+ v = 0,$$

we obtain the commutation relations

$$[u(x), u(y)]_{x^- = y^-} = -\frac{i}{4} \epsilon(x^+ - y^+) \tag{13}$$

$$[v(x), v(y)]_{x^+ = y^+} = -\frac{i}{4} \epsilon(x^- - y^-)$$

where $\epsilon(x)$ is usual alternating function. Equations of motion(12) and the commutation relation (13) enable one to get the plane wave solution of $u(x)$ and $v(x)$

$$u(x^+) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{0} dp \frac{1}{\sqrt{|p|}} \left[e^{ipx^+} a(p) + e^{-ipx^+} a^\dagger(p) \right] \tag{14}$$

$$\tag{15}$$

$$v(x^-) = \frac{1}{2\sqrt{\pi}} \int_0^{\infty} dp \frac{1}{\sqrt{p}} \left[e^{ipx^-} a^\dagger(p) + e^{-ipx^-} a(p) \right]$$

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and the second quantization rules

$$[a(p), a^\dagger(q)] = \delta(p - q) \tag{16}$$

$$[a(p), a(q)] = [a^\dagger(p), a^\dagger(q)] = 0.$$

By inserting the plane wave solutions (14) to Eq.(11) the Poincaré charges become

$$P^+ = \frac{1}{2} \int_0^\infty dp\, p[a^\dagger(p)a(p) + a(p)a^\dagger(p)] \tag{17}$$

$$P^- = \frac{1}{2} \int_{-\infty}^0 dp \mid p \mid [a^\dagger(p)a(p) + a(p)a^\dagger(p)].$$

From Eq.(16) we can obtain the Hamiltonian and momentum operators derived in usual Minkowski space. This is the evidence of the isomorphism between the light-cone quantized theory and the equal-time quantized theory commented in Ref.[13]. In order to obtain the propagators we define the Green's function

$$g(x - y) = i < 0 \mid T^* \phi(x)\phi(y) \mid 0 > . \tag{18}$$

where T^*-ordering is taken with respect to the x^+ coordinate for v and x^- coordinate for u. If one uses the integral representation of the step function

$$\theta(x) = \frac{1}{2\pi i} \int_{-\infty}^\infty d\tau \frac{e^{ix\tau}}{\tau - i\epsilon}, \tag{19}$$

the direct calculation gives

$$g(x) = -\frac{2}{(2\pi)^2} \int_{-\infty}^\infty dp^+ \int_{-\infty}^\infty dp^- \frac{1}{2p^+p^- + i\epsilon} e^{i(p^+x^- + p^-x^+)}. \tag{20}$$

So the modification term appeared in Eq.(1) disappears from the propagator. This result seems natural since the light-cone quantization procedure formulated in this way gives a Causal-prescription in four-dimensional

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light-cone gauge theories[7] and in the two-dimensional massless case the Causal-prescription is manifestly usual ML- prescription.

For the completeness let us consider the fermionic case too. For fermionic case the equations of motion are

$$\partial_+ \psi_+ = 0 \tag{21}$$

$$\partial_- \psi_- = 0$$

where γ matrices are taken as

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \gamma^5 = \gamma^0 \gamma^1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{22}$$

$$\gamma^+ = \sqrt{2} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \gamma^- = \sqrt{2} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

Since the null-plane formalism gives a quantization rule

$$\{\psi_+^*(x), \psi_+(y)\}_{x^+=y^+} = \delta(x^- - y^-) \tag{23}$$

$$\{\psi_-^*(x), \psi_-(y)\}_{x^-=y^-} = \delta(x^+ - y^+), \tag{24}$$

one can derive the plane-wave solutions

$$\psi_+(x^-) = \frac{1}{\sqrt{2\pi}} \int_0^\infty dk \left[a(k)e^{-ikx^-} + b^\dagger(k)e^{ikx^-} \right] \tag{25}$$

$$\psi_-(x^+) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 dk \left[a(k)e^{-ikx^+} + b^\dagger(k)e^{ikx^+} \right] \tag{26}$$

,which is already used in Ref.[13] when proving the isomorphism between the light-cone and equal-time quantized theories, and the second quantization rules

$$\{a(p), a^\dagger(q)\} = \{b(p), b^\dagger(q)\} = \delta(p - q). \tag{27}$$

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By defining the Green's function

$$g_{\alpha\beta}(x - y) = i < 0 \mid T^* \psi_\alpha(x) \bar{\psi}_\beta(y) \mid 0 >,\tag{28}$$

the direct calculation gives

$$g(x - y) = \frac{\sqrt{2}}{(2\pi)^2} \int_{-\infty}^{\infty} dk^+ dk^- \frac{\gamma^\mu k_\mu}{k^2 + i\epsilon} e^{ik(x-y)}\tag{29}$$

which is the usual Causal-prescription Green's function. So it is shown that the modification term in two-dimensional massless propagators can be cancelled by applying the light-cone quantization procedure recently developed and introducing the two distinct null planes. The appearance and the disappearanec of the modification term is closely related with a recent debate between ML- and PV-prescriptions which occured in four-dimensional light-cone gauge theory. As was stated previously, the disappearance of the modification term by introducing the two distinct null planes does not guarantee that ML-prescription is more consistent than PV-prescription. To determine which is more consistent in two-dimensional theories it is important to calculate the VEV of the Wilson-loop operator defined at the null planes as suggested before. This will be reported elsewhere.

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[12] My conventions are
$$x^\pm = \tfrac{1}{\sqrt{2}}(x^0 \pm x^1) \qquad g^{++} = g^{--} = 0 \qquad g^{+-} = g^{-+} = 1$$

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